Fachhochschule Südwestfalen
University of Applied Sciences

Predictive Analytics

Ch11. Advanced forecasting methods

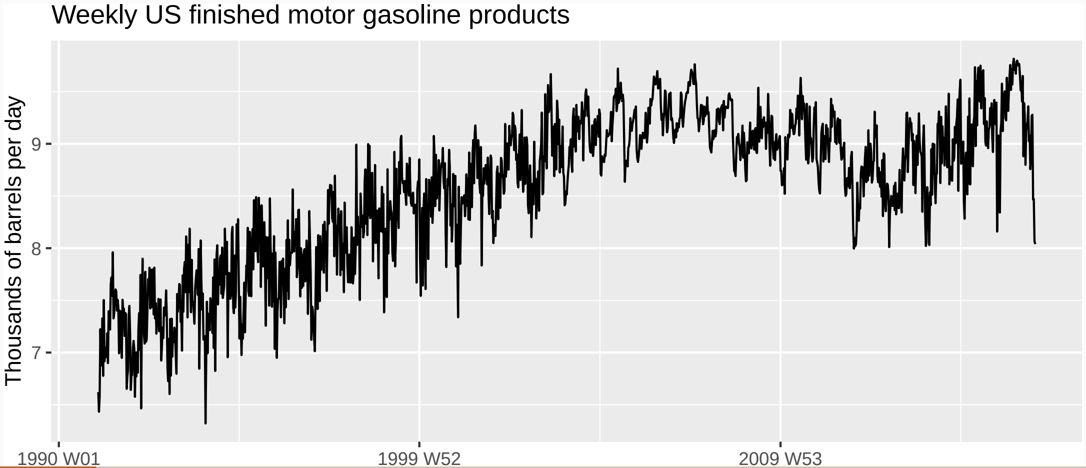
Prof. Dr. Benjamin Buchwitz

Wir geben Impulse

Outline

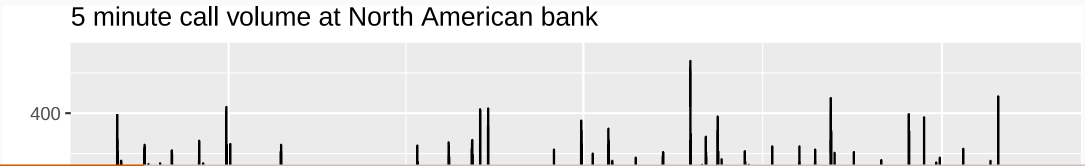
Examples

```r
us_gasoline %>% autoplot(Barrels) +
  labs(x = "Year", y = "Thousands of barrels per day",
       title = "Weekly US finished motor gasoline products")
```



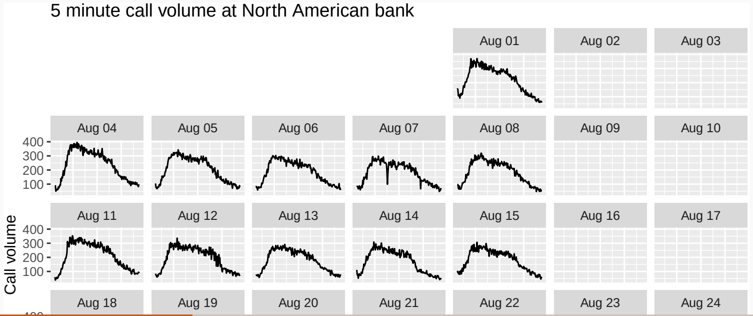
Examples

```
calls <- read_tsv("http://robjhyndman.com/data/callcenter.txt") %>%
  rename(time = `...1`) %>%
  pivot_longer(-time, names_to="date", values_to="volume") %>%
  mutate(
    date = as.Date(date, format = "%d/%m/%Y"),
    datetime = as_datetime(date) + time
  ) %>%
  as_tsibble(index = datetime)
calls %>%
  fill_gaps() %>%
  autoplot(volume) +
  labs(x = "Weeks", y = "Call volume",
       title = "5 minute call volume at North American bank")
```



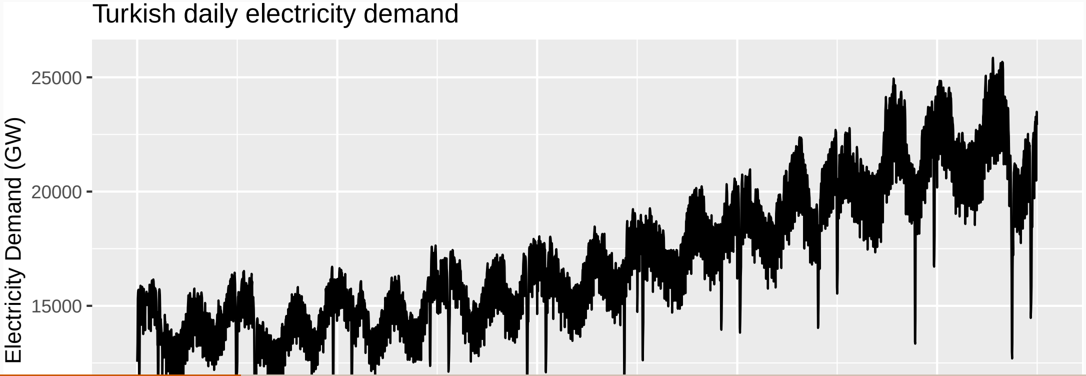
Examples

```
library(sugrrants)
calls %>%
  filter(yearmonth(date) == yearmonth("2003 August")) %>%
  ggplot(aes(x = time, y = volume)) +
  geom_line() +
  facet_calendar(date) +
  labs(x = "Weeks", y = "Call volume",
       title = "5 minute call volume at North American bank")
```



Examples

```r
turkey_elec <- read_csv("data/turkey_elec.csv", col_names = "Demand") %>%
  mutate(Date = seq(ymd("2000-01-01"), ymd("2008-12-31"), by = "day")) %>%
  as_tsibble(index = Date)
turkey_elec %>% autoplot(Demand) +
  labs(title = "Turkish daily electricity demand",
       x = "Year", y = "Electricity Demand (GW)")
```



TBATS

Trigonometric terms for seasonality
Box-Cox transformations for heterogeneity
ARMA errors for short-term dynamics
Trend (possibly damped)
Seasonal (including multiple and non-integer periods)

TBATS model

$$y_t = \text{observation at time } t$$

$$y_t^{(\omega)} = \begin{cases} (y_t^\omega - 1)/\omega & \text{if } \omega \neq 0; \\ \log y_t & \text{if } \omega = 0. \end{cases}$$

$$y_t^{(\omega)} = \ell_{t-1} + \phi b_{t-1} + \sum_{i=1}^{M} s_{t-m_i}^{(i)} + d_t$$

$$\ell_t = \ell_{t-1} + \phi b_{t-1} + \alpha d_t$$

$$b_t = (1-\phi)b + \phi b_{t-1} + \beta d_t$$

$$d_t = \sum_{i=1}^{p} \phi_i d_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t$$

$$s_t^{(i)} = \sum_{j=1}^{k_i} s_{j,t}^{(i)} \qquad \begin{aligned} s_{j,t}^{(i)} &= s_{j,t-1}^{(i)} \cos \lambda_j^{(i)} + s_{j,t-1}^{*(i)} \sin \lambda_j^{(i)} + \gamma_1^{(i)} d_t \\ s_{j,t}^{(i)} &= -s_{j,t-1}^{(i)} \sin \lambda_j^{(i)} + s_{j,t-1}^{*(i)} \cos \lambda_j^{(i)} + \gamma_2^{(i)} d_t \end{aligned}$$

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global and local trend

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ARMA error

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$$s_t^{(i)} = \sum_{j=1}^{k_i} s_{j,t}^{(i)}$$

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Fourier-like seasonal terms

$$s_{j,t}^{(i)} = -s_{j,t-1}^{(i)} \sin \lambda_j^{(i)} + s_{j,t-1}^{*(i)} \cos \lambda_j^{(i)} + \gamma_2^{(i)} d_t$$

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Box-Cox transformation

TBATS

Trigonometric

Box-Cox

ARMA

Trend

Seasonal

$y_t^{(\omega)} = \ell_{t-}$ ... t

M seasonal periods

$\ell_t = \ell_{t-}$ ...

$b_t = (1$ ...

global and local trend

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Fourier-like seasonal terms

$$s_{j,t}^{(i)} = -s_{j,t-1}^{(i)} \sin\lambda_j^{(i)} + s_{j,t-1}^{*(i)} \cos\lambda_j^{(i)} + \gamma_2^{(i)} d_t$$

Complex seasonality

```r
gasoline %>% tbats() %>% forecast() %>% autoplot()
```

Complex seasonality

```
calls %>% tbats() %>% forecast() %>% autoplot()
```

Complex seasonality

```
telec %>% tbats() %>% forecast() %>% autoplot()
```

TBATS

Trigonometric terms for seasonality

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ARMA errors for short-term dynamics

Trend (possibly damped)

Seasonal (including multiple and non-integer periods)

- Handles non-integer seasonality, multiple seasonal periods.
- Entirely automated
- Prediction intervals often too wide
- Very slow on long series

Outline

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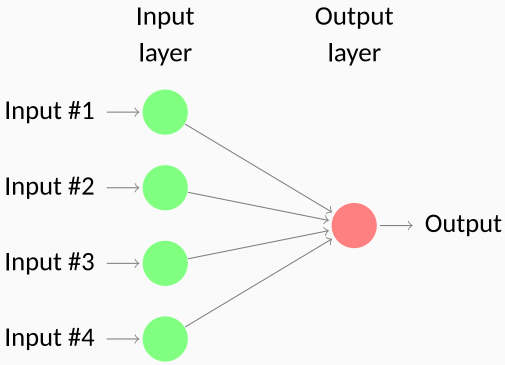
Neural network models

Simplest version: linear regression



Neural network models

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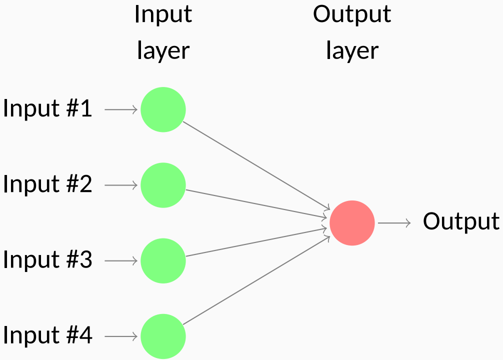
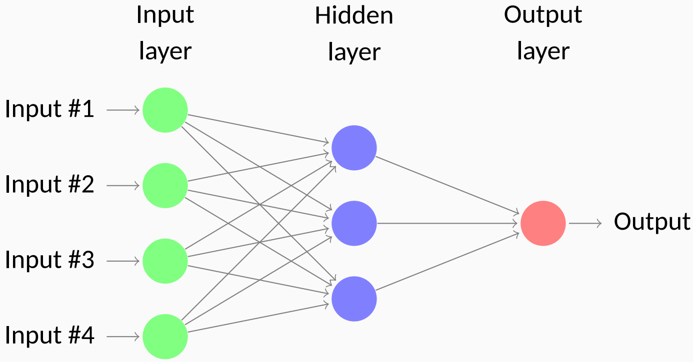


- Coefficients attached to predictors are called "weights".
- Forecasts are obtained by a linear combination of inputs.
- Weights selected using a "learning algorithm" that minimises a "cost function".

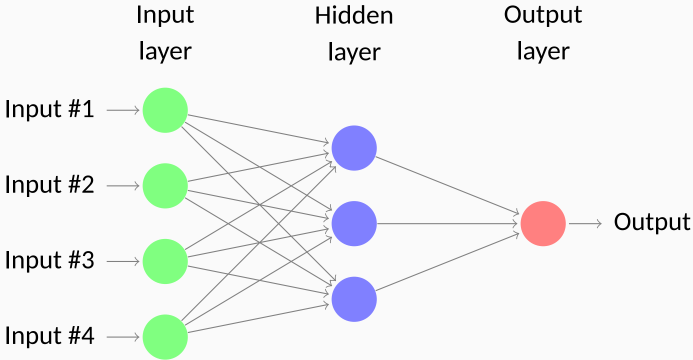
Neural network models

Nonlinear model with one hidden layer



Neural network models

Nonlinear model with one hidden layer



- A **multilayer feed-forward network** where each layer of nodes receives inputs from the previous layers.
- Inputs to each node combined using linear combination.
- Result modified by nonlinear function before being output.

Neural network models

Inputs to hidden neuron j linearly combined:

$$z_j = b_j + \sum_{i=1}^{4} w_{i,j} x_i.$$

Modified using nonlinear function such as a sigmoid:

$$s(z) = \frac{1}{1 + e^{-z}},$$

This tends to reduce the effect of extreme input values, thus making the network somewhat robust to outliers.

Neural network models

- Weights take random values to begin with, which are then updated using the observed data.
- There is an element of randomness in the predictions. So the network is usually trained several times using different random starting points, and the results are averaged.
- Number of hidden layers, and the number of nodes in each hidden layer, must be specified in advance.

NNAR models

- Lagged values of the time series can be used as inputs to a neural network.
- NNAR(p, k): p lagged inputs and k nodes in the single hidden layer.
- NNAR(p, 0) model is equivalent to an ARIMA(p, 0, 0) model but without stationarity restrictions.
- Seasonal NNAR(p, P, k): inputs $(y_{t-1}, y_{t-2}, \dots, y_{t-p}, y_{t-m}, y_{t-2m}, y_{t-Pm})$ and k neurons in the hidden layer.
- NNAR(p, P, $0)_m$ model is equivalent to an ARIMA(p, 0, 0)(P,0,0$)_m$ model but without stationarity restrictions.

NNAR models in R

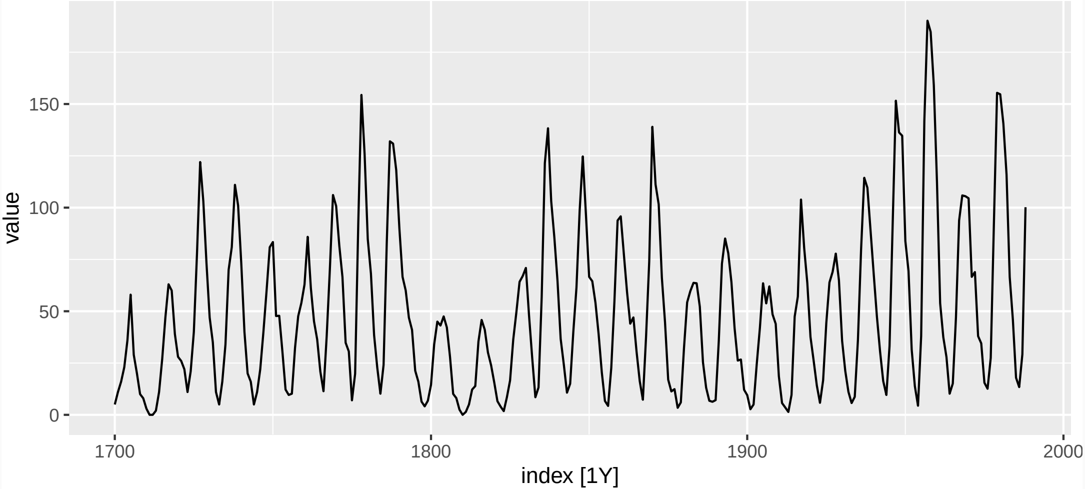
- The `nnetar()` function fits an NNAR$(p, P, k)_m$ model.
- If p and P are not specified, they are automatically selected.
- For non-seasonal time series, default $p =$ optimal number of lags (according to the AIC) for a linear AR(p) model.
- For seasonal time series, defaults are $P = 1$ and p is chosen from the optimal linear model fitted to the seasonally adjusted data.
- Default $k = (p + P + 1)/2$ (rounded to the nearest integer).

Sunspots

- Surface of the sun contains magnetic regions that appear as dark spots.
- These affect the propagation of radio waves and so telecommunication companies like to predict sunspot activity in order to plan for any future difficulties.
- Sunspots follow a cycle of length between 9 and 14 years.

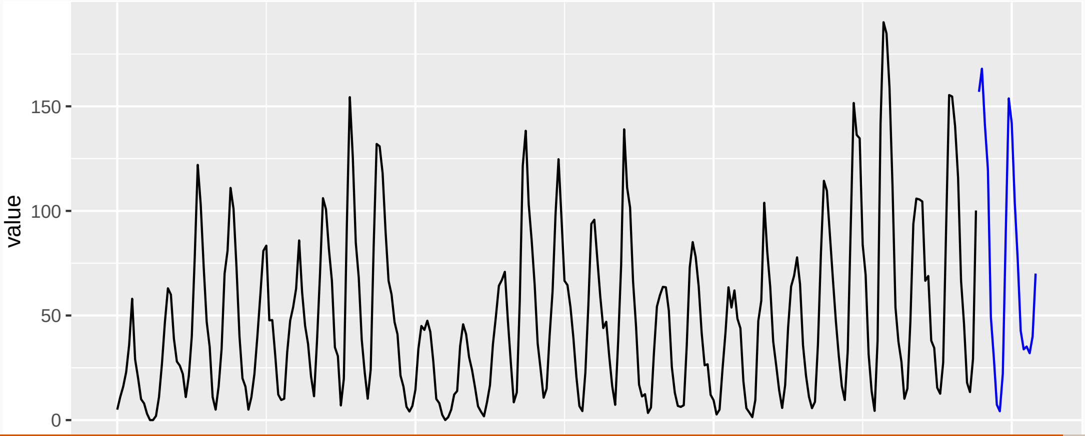
Sunspots

```
sunspots <- sunspot.year %>% as_tsibble()
sunspots %>% autoplot(value)
```



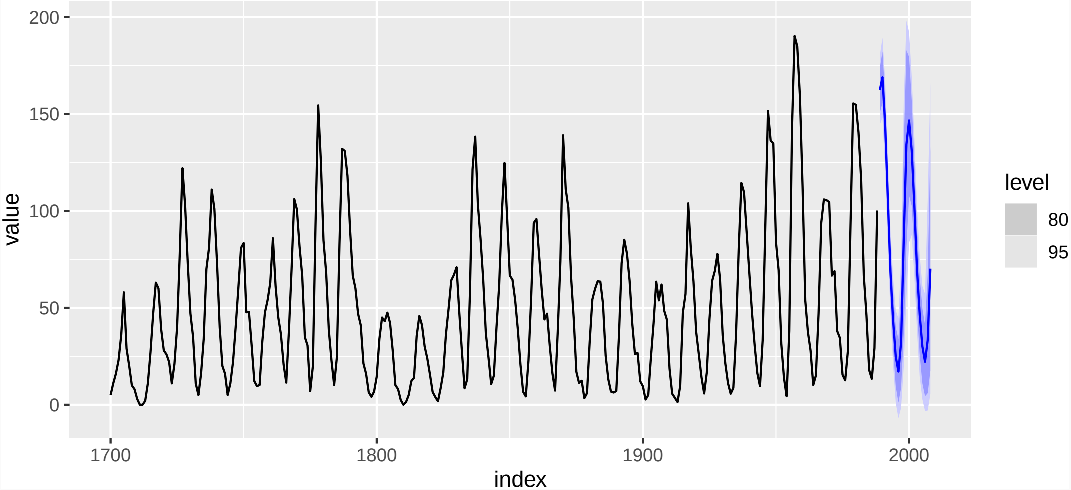
NNAR(9,5) model for sunspots

```
sunspots <- sunspot.year %>% as_tsibble()
fit <- sunspots %>% model(NNETAR(value))
fit %>% forecast(h=20, times = 1) %>%
  autoplot(sunspots, level = NULL)
```



Prediction intervals by simulation

```
fit %>% forecast(h=20) %>%
  autoplot(sunspots)
```



Outline